EARTHQUAKE ENGINEERING AND STRUCTURAL DYNAMICS, VOL. 26, 1125–1136 (1997)

EFFECT OF WEIGHTING A STROKE OF AN ACTIVE MASS DAMPER IN THE LINEAR QUADRATIC REGULATOR PROBLEM

YOSHIKI IKEDA*

Senior Research Engineer, Kobori Research Complex, Kajima Corporation, KI Bldg. 6-5-30 Akasaka, Minato-ku, Tokyo 107, Japan

SUMMARY

The effect of weighting on the linear quadratic regulator problem is analytically described for a single-degree-of-freedom undamped structure controlled by an Active Mass Damper (AMD). The feedback control law is introduced in a closed form after both the state equation and the performance index are non-dimensionalized. The closed expression proves that the feedback gain associated with a stroke of the AMD depends only on its weighting parameter and the mass and frequency ratios of the AMD to the structure, and the gain becomes zero when the stroke is neglected in weighting. Weighting the stroke apparently increases the AMD damping with increasing natural frequency. This pole allocation leads to ineffective control, which compels the auxiliary mass to be fixed to the structure. The essential feedback gain associated with the structural velocity changes its sign according to the weighting state. The effect of tuning the AMD to the structural natural frequency can be seen when the stroke is weighted a little. From a viewpoint of stability, the optimal weighting parameter for the stroke is proposed when an active tuned mass damper is installed.

Earthquake Engng. Struct. Dyn., **26**, 1125–1136 (1997)

No. of Figures: 8. No. of Tables: 0. No. of References: 15.

KEY WORDS: active control; active mass damper; closed expression; linear quadratic regulator; optimal weighting parameter; stability

1. INTRODUCTION

The Active Mass Damper (AMD) is widely recognized as one of the most advantageous strategies in active/hybrid vibration control of civil engineering structures. It is well known that approximately 20 AMDs have already been applied to full-scale civil engineering structures.[1,2] In most of these applications,[3–7] feedback gains are changed on-line by sensing the vibration level, because the response levels of both the structure and the AMD vary many times under a non-stationary input excitation such as an earthquake and a wind. When the feedback gain to be selected properly is designed *a priori* on the Linear Quadratic Regulator (LQR) control theory, the AMD stroke is often considered in weighting.[6,8,9] This is because practical implementation physically limits the space available for an AMD when an input excitation becomes large. In addition, the LQR theory can be easily applied to the problem with constraint on the stroke even if an input disturbance is unknown beforehand. Recently, based on the LQR problem, Nagashima *et al.*[9] proposed a practical gain schedule method which continuously changes the feedback gain to constrain the response of the AMD. However, the effect of weighting the stroke on the LQR problem has not yet been described analytically, though some proposed gain schedule methods have been confirmed to be effective in both numerical simulation and experimental and practical applications. The guideline for weighting the stroke has been unclear as yet due to a weak point of the LQR problem.

* Correspondence to: Yoshiki Ikeda, Senior Research Engineer, Kobori Research Complex, Kajima Corporation, KI Bldg. 6-5-30 Akasaka, Minato-ku, Tokyo 107, Japan. E-mail: ikeda@krc.kajima.co.jp

CCC 0098–8847/97/111125–12$17·50

Received 2 December 1996
Revised 19 May 1997

In solving the LQR problem, both the Potter method[10] and the Laub method[11] are widely recognized as the most useful and standard procedure to obtain the stationary feedback gain, especially for actively controlling the MDOF structure. However, either procedure is unsatisfactory in discussing the essential properties of active control, because it is like a 'black box' in the way it handles the variety of design parameters. To apply the intrinsic properties to the rule-base control with fuzzy logic, Abe introduced an approximate closed expression for a single-degree-of-freedom (SDOF) undamped structure with an active tuned mass damper (ATMD).[12] This solution is obtained by retaining the lowest-order term in the singular perturbation method. Then, the physical property is described in detail when the weighting parameter with respect to a control force becomes infinite. This assumption corresponds to the case in which the control force is small compared to the excitation force. Now, the precise closed solution is required to research the AMD system in a broad weighting range on the LQR problem.

This paper analytically describes the effect of weighting in the LQR problem when an SDOF undamped structure is controlled by an AMD. To describe the essential properties, the state equation and the performance index are transformed according to the non-dimensionalization proposed by Mitsuta and Seto.[13] The closed expression of the stationary feedback control law proves that the feedback gain associated with the AMD stroke depends only on its weighting parameter and the mass and frequency ratios of the damper to the structure, and the gain becomes zero when the stroke is neglected in weighting. From a broad viewpoint, weighting the stroke apparently makes the damping of the AMD higher with a higher natural frequency, while it makes the structural damping decrease with a nearly constant frequency. This pole allocation leads to ineffective control, which compels the auxiliary mass to become fixed to the structure. The essential feedback gain associated with the structural velocity changes its sign according to the weighting state. The effect of tuning the AMD to the frequency of the installed structure can be seen when the stroke is weighted a little.

When the AMD is tuned to the structure and the velocity of the structure is constantly weighted, it is recognized that the damping ratio of the structural mode apparently becomes highest at a certain small weighting parameter with respect to the AMD stroke. This weighting parameter makes the system stability higher and advantageously suppresses both the stroke and the control force better than when the ATMD stroke is neglected in weighting. The dynamic property proposes the optimal weighting parameter with respect to the stroke for designing the ATMD.

2. FEEDBACK GAIN IN A CLOSED EXPRESSION

2.1. Non-dimensional formulation

When an SDOF undamped structure is controlled by an AMD as shown in Figure 1, the state equation is expressed as

$$\begin{Bmatrix} \ddot{x}_A(t) \\ \ddot{x}_B(t) \\ \dot{x}_A(t) \\ \dot{x}_B(t) \end{Bmatrix} = \begin{bmatrix} -2(1+\mu)h_A\omega_A & 0 & -(1+\mu)\omega_A^2 & \omega_B^2 \\ 2\mu h_A\omega_A & 0 & \mu\omega_A^2 & -\omega_B^2 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \dot{x}_A(t) \\ \dot{x}_B(t) \\ x_A(t) \\ x_B(t) \end{Bmatrix} + \begin{Bmatrix} -((1/\mu)+1) \\ 1 \\ 0 \\ 0 \end{Bmatrix} \frac{u(t)}{m_B} \tag{1}$$

where ω_B and m_B are the uncoupled natural circular frequency and the mass of the structure, respectively; ω_A and h_A are the uncoupled natural circular frequency and the damping ratio of the AMD, respectively; μ is the mass ratio of the AMD to the structure; $x_B(t)$ is the structure displacement to the base; $x_A(t)$ is the stroke of the AMD; and $u(t)$ is the control force. The structural damping is neglected in this formulation, because it becomes inessential to the physical properties when control adds damping to a structure.

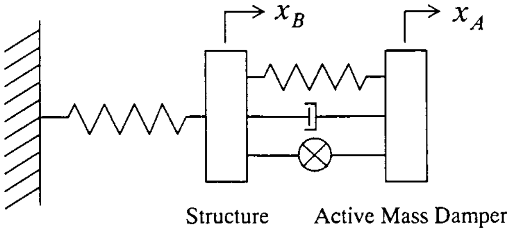

Figure 1. SDOF undamped structure with Active Mass Damper

By assuming that the weighting matrix with respect to the state vector is a diagonal one, we obtain the performance index that is minimized for the LQR problem, that is

$$J=\frac{1}{2}\int_0^{\infty}\left[\begin{Bmatrix}\dot{x}_A(t)\\ \dot{x}_B(t)\\ x_A(t)\\ x_B(t)\end{Bmatrix}^T\begin{bmatrix}q_{11}&0&0&0\\0&q_{22}&0&0\\0&0&q_{33}&0\\0&0&0&q_{44}\end{bmatrix}\begin{Bmatrix}\dot{x}_A(t)\\ \dot{x}_B(t)\\ x_A(t)\\ x_B(t)\end{Bmatrix}+ru(t)^2\right]\mathrm{d}t \tag{2}$$

where the weighting parameters q_{11}, q_{22}, q_{33}, q_{44} and r are positive.

According to the non-dimensionalization proposed by Mitsuta and Seto[13], equations (3a) and (3b) transform the state equation and the performance index as equations (4) and (5), respectively.

$$\tau=\omega_B t,\quad \bar{u}(\tau)=u(t)/(m_B\omega_B^2),\quad \gamma=\omega_A/\omega_B \tag{3a}$$

$$\bar{q}_{11}=\omega_B^2 q_{11},\quad \bar{q}_{22}=\omega_B^2 q_{22},\quad \bar{q}_{33}=q_{33},\quad \bar{q}_{44}=q_{44},\quad \bar{r}=(m_B\omega_B^2)^2 r \tag{3b}$$

$$\begin{Bmatrix}\ddot{z}_A(\tau)\\ \ddot{z}_B(\tau)\\ \dot{z}_A(\tau)\\ \dot{z}_B(\tau)\end{Bmatrix}=\begin{bmatrix}-2(1+\mu)h_A\gamma&0&-(1+\mu)\gamma^2&1\\2\mu h_A\gamma&0&\mu\gamma^2&-1\\1&0&0&0\\0&1&0&0\end{bmatrix}\begin{Bmatrix}\dot{z}_A(\tau)\\ \dot{z}_B(\tau)\\ z_A(\tau)\\ z_B(\tau)\end{Bmatrix}+\begin{Bmatrix}-((1/\mu)+1)\\1\\0\\0\end{Bmatrix}\bar{u}(\tau) \tag{4}$$

$$\bar{J}=\frac{1}{2}\int_0^{\infty}\left[\begin{Bmatrix}\dot{z}_A(\tau)\\ \dot{z}_B(\tau)\\ z_A(\tau)\\ z_B(\tau)\end{Bmatrix}^T\begin{bmatrix}\bar{q}_{11}/\bar{r}&0&0&0\\0&\bar{q}_{22}/\bar{r}&0&0\\0&0&\bar{q}_{33}/\bar{r}&0\\0&0&0&\bar{q}_{44}/\bar{r}\end{bmatrix}\begin{Bmatrix}\dot{z}_A(\tau)\\ \dot{z}_B(\tau)\\ z_A(\tau)\\ z_B(\tau)\end{Bmatrix}+\bar{u}(\tau)^2\right]\mathrm{d}\tau \tag{5}$$

This transformation describes that the feedback gain g_i subjected to equations (1) and (2) can be obtained from the non-dimensional gain $\bar{g}_i$ as follows:

$$g_1=\omega_B m_B\bar{g}_1,\quad g_2=\omega_B m_B\bar{g}_2,\quad g_3=\omega_B^2 m_B\bar{g}_3\quad\text{and}\quad g_4=\omega_B^2 m_B\bar{g}_4 \tag{6a}$$

where

$$u(t)=g_1\dot{x}_A(t)+g_2\dot{x}_B(t)+g_3x_A(t)+g_4x_B(t) \tag{6b}$$

$$\bar{u}(\tau)=\bar{g}_1\dot{z}_A(\tau)+\bar{g}_2\dot{z}_B(\tau)+\bar{g}_3z_A(\tau)+\bar{g}_4z_B(\tau) \tag{6c}$$

2.2. Feedback control law

By directly solving the Riccati-type matrix equation, we introduce the feedback control law in the closed form as follows:

$$\bar{g}_1 = -2\mu h_{\mathrm{A}}\gamma + \sqrt{4\mu^2 h_{\mathrm{A}}^2\gamma^2 + 2\lambda + \bar{q}_{11}/\bar{r}} \tag{7a}$$

$$\bar{g}_2 = \pm\sqrt{(1+\mu)\left\{1 + \bar{q}_{44}/\bar{r} - \frac{(\mu\gamma^2 + \lambda/\mu)^2}{\mu^2\gamma^4 + \bar{q}_{33}/\bar{r}}\right\} + \frac{2(\mu\gamma^2 + \lambda/\mu)}{\sqrt{\mu^2\gamma^4 + \bar{q}_{33}/\bar{r}}} - 2 + \bar{q}_{22}/\bar{r}} \tag{7b}$$

$$\bar{g}_3 = -\mu\gamma^2 + \sqrt{\mu^2\gamma^4 + \bar{q}_{33}/\bar{r}} \tag{7c}$$

$$\bar{g}_4 = 1 - \frac{\mu\gamma^2 + \lambda/\mu}{\sqrt{\mu^2\gamma^4 + \bar{q}_{33}/\bar{r}}} \tag{7d}$$

The feedback gains relate only to λ which satisfies the fourth-ordered algebraic equation as

$$\lambda^4 + b_1\lambda^3 + b_2\lambda^2 + b_3\lambda + b_4 = 0 \tag{8a}$$

where

$$\begin{aligned}
b_1 &= 4\{(1+\mu)c_3 + \mu^2\gamma^2\}\\
b_2 &= 4(1+\mu)^2c_3^2 - 16\mu c_3^{\frac{3}{2}} + 2c_3\{2(1+\mu)(2\mu^2\gamma^2 + c_1) + \mu(2\bar{g}_3 - \mu c_4)\} + 6\mu^4\gamma^4\\
b_3 &= -4\mu c_3^{\frac{2}{3}}\{(1+\mu)(2\bar{g}_3 + \mu c_4) + 2\mu c_2\} - 8\mu c_3^{\frac{3}{2}}(2\mu^2\gamma^2 + c_1)\\
&\quad + 4\mu^2 c_3\{(1+\mu)(\mu^2\gamma^2 + 2c_1)\gamma^2 + \mu\gamma^2(2\bar{g}_3 - \mu c_4)\} + 4\mu^6\gamma^6\\
b_4 &= \{\mu c_3(2\bar{g}_3 - \mu c_4) + \mu^4\gamma^4\}^2 - 4\mu^2 c_1 c_3^2 c_5\\
c_1 &= 4\mu^2 h_{\mathrm{A}}^2\gamma^2 + \bar{q}_{11}/\bar{r}, \quad c_2 = -2 + \bar{q}_{22}/\bar{r}, \quad c_3 = \mu^2\gamma^4 + \bar{q}_{33}/\bar{r}, \quad c_4 = 1 + \bar{q}_{44}/\bar{r}\\
c_5 &= (1+\mu)(c_4 - \mu^2\gamma^4/c_3) + 2\mu\gamma^2/\sqrt{c_3} + c_2
\end{aligned} \tag{8b}$$

Both the appropriate λ and the alternative sign in equation (7b) must be determined by considering real gains and system stability.

Equation (7c) proves that the gain $\bar{g}_3$ associated with the stroke relates only to μ, γ and $\bar{q}_{33}/\bar{r}$ independently of equation (8a), and it becomes zero when the stroke is neglected in weighting. If $\gamma = 1{\cdot}0$ is assumed, equation (7c) completely corresponds to the gain for the ATMD stroke which is approximately obtained by the singular perturbation method.[12] When the non-dimensional natural frequency in the ith mode is defined as γ_i, the non-negative $\bar{g}_3$ introduces $\gamma_1\gamma_2 \geqslant \gamma$ for the pole allocation. Even if the structural damping is considered, both $\bar{g}_3$ and $\bar{g}_4$ may be led in the same expression.

The alternative sign in equation (7b) implies that the essential feedback gain associated with the structural velocity may change its sign corresponding to the weighting state. When the gain become zero, λ is expressed as

$$\lambda = \frac{\mu\sqrt{\mu^2\gamma^4 + \bar{q}_{33}/\bar{r}}}{1+\mu}\left[1 + \sqrt{1 + (1+\mu)\{(1+\mu)(1 + \bar{q}_{44}/\bar{r}) + \bar{q}_{22}/\bar{r} - 2\}}\right] - \mu^2\gamma^2 \tag{9}$$

3. EFFECT OF WEIGHTING A STROKE

3.1. Effect of weighting a stroke

This section describes the effect of weighting the stroke, comparing the ATMD with the AMD. The mass ratio of the AMD to the structure μ is assumed to be 0·01 by considering the practical applications. When the

AMD is tuned to the frequency of the installed structure, both the frequency ratio and the damping ratio of the AMD comply with the optimization for a tuned mass damper (TMD) proposed by Den Hartog,[14] that is

$$\gamma = \frac{1}{1+\mu} \quad \text{and} \quad h_{\mathrm{A}} = \sqrt{\frac{3\mu}{8(1+\mu)^3}} \tag{10}$$

Thus, the frequency ratio γ is 0·9901 and the damping ratio h_{A} is 6·033 per cent. This damper is henceforth referred to as an ATMD which is a conversion of the passive-type TMD into an active damper.[15] When the damper is not tuned, it is referred to as an AMD, the frequency ratio is assumed to be 0·2 and the damping ratio is zero. The weighting parameter $\bar{q}_{22}/\bar{r}$ with respect to the structural velocity is maintained at a certain value, because the parameter is the most essential to the LQR problem. The weighting parameters with respect to both the velocity of the damper and the structural displacement are neglected in numerical examples.

Figure 2(a) shows the dependency of four feedback gains on the weighting parameter $\bar{q}_{33}/\bar{r}$ with respect to the stroke under the condition of $\bar{q}_{22}/\bar{r} = 0{\cdot}1$. The feedback gain $\bar{g}_1$ associated with the velocity of the damper slowly increases with the positive sign, depending strongly on λ. This is because the design parameters attach great importance to the second term in the square root in equation (7a). The gain $\bar{g}_2$ for the structural velocity starts at $\bar{q}_{33}/\bar{r} = 0$ with the negative sign and changes its sign soon afterward, and this becomes the important property in designing the controller. This gain varies remarkably in the small weighting range. $\bar{g}_3$ for the stroke constantly increases with the positive sign according to equation (7c). $\bar{g}_4$ for the structural displacement remains negative, though it changes the sign of the gradient. Figure 2(b) shows the non-dimensional natural frequencies of the controlled system, corresponding to Figure 2(a). The weighting influences the damper's natural frequency more strongly than the structure's one. From a broad viewpoint, weighting the stroke apparently increases the natural frequency of the damper and keeps the structural frequency nearly constant. Figure 2(c) shows the damping ratios of the controlled system, corresponding to Figures 2(a) and 2(b). The damping ratios of both the ATMD and the AMD finally approach about 70 per cent with decrease in structural damping. The AMD's damping ratio approaches this constant value more quickly than the ATMD's does. The tuning of the AMD affects the pole allocation when the weighting parameter is relatively small. The pole allocation shown in Figures 2(b) and 2(c) causes the ineffective control, which compels the auxiliary mass to become fixed to the structure. This corresponds to minimizing the frequency transfer function of the mass damper to the structure. The same tendency is generally recognized even if the weighting parameter with respect to the structural velocity is changed.

Figure 3 magnifies Figure 2 in the weighting range near $\bar{q}_{33}/\bar{r} = 0$. Figure 3(a) closes up two feedback gains with respect to the structural response. When the stroke is neglected in weighting, the essential gain $\bar{g}_2$ with respect to the structural velocity is $-0{\cdot}0615$ for the ATMD and $-0{\cdot}316$ for the AMD. Then, the gain $\bar{g}_4$ with respect to the structural displacement is $-0{\cdot}299$ for the ATMD and $-0{\cdot}009$ for the AMD. $\bar{g}_2$ changes the negative sign to the positive at $\bar{q}_{33}/\bar{r} = 0{\cdot}330 \times 10^{-4}$ for the ATMD, and at $\bar{q}_{33}/\bar{r} = 1{\cdot}223 \times 10^{-4}$ for the AMD. At these weighting points, according to equation (9), λ becomes $0{\cdot}502 \times 10^{-4}$ for the ATMD and $1{\cdot}404 \times 10^{-4}$ for the AMD. The closed-up figure confirms the difference between the ATMD and the AMD in the small weighting range which may be useful for practical application. Figure 3(b) illustrates that the AMD makes the two natural frequencies encounter each other at $\bar{q}_{33}/\bar{r} = 0{\cdot}879 \times 10^{-4}$. The initial non-dimensional frequencies are 0·817 and 1·212 when the ATMD is applied, and 0·200 and 1·002 when the AMD is applied. These values satisfy the equality $\gamma_1\gamma_2 = \gamma$, which is introduced from the pole allocation. With its similarity to Figure 3(b), Figure 3(c) illustrates the differences between the ATMD and the AMD especially in the damper's mode. The damping ratio of the ATMD varies relatively slowly from the initial value of 23·9 per cent, while the damping ratio of the AMD varies faster from the initial value of 3·4 per cent. However, the structural damping is slightly different. The ATMD changes the structural damping ratio from 16·2 to 6·8 per cent, forming a low peak of 17·9 per cent at

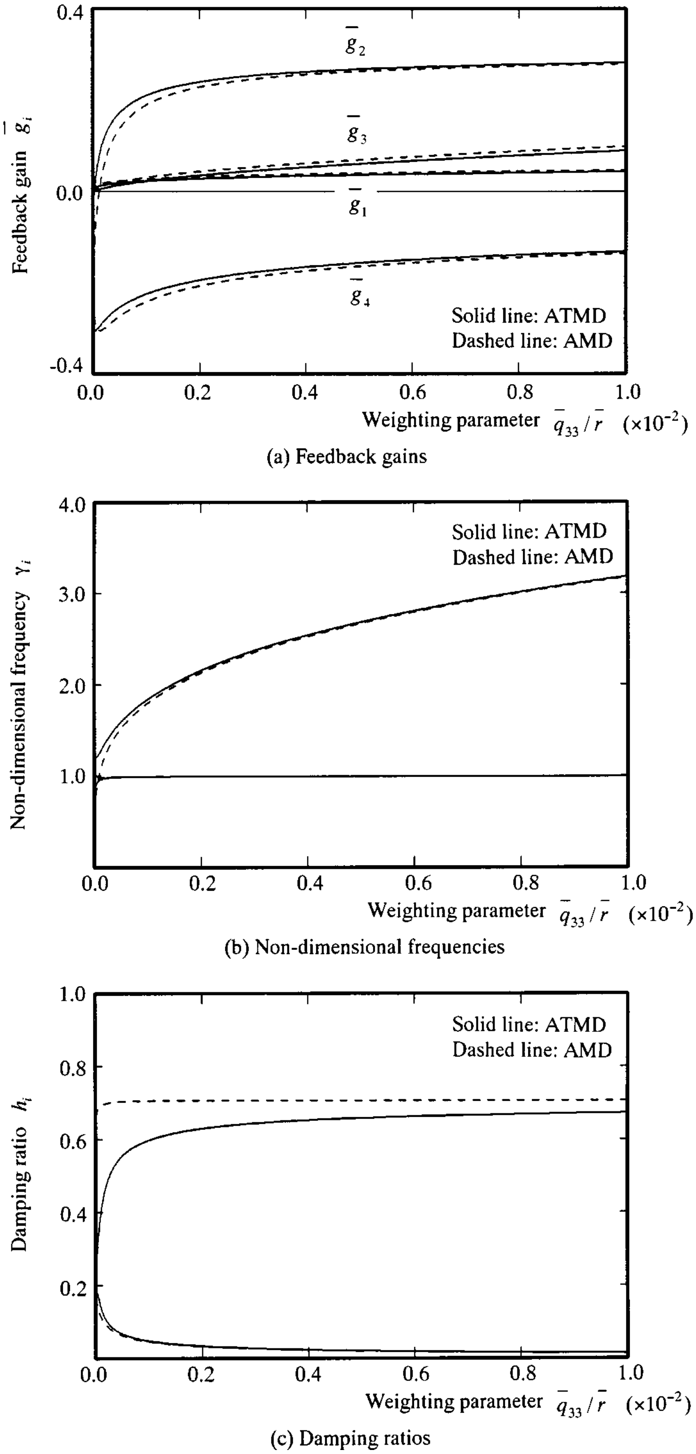

Figure 2. Feedback gains and poles depending on the weighting parameter $\bar{q}_{33}/\bar{r}$ with respect to the stroke under the condition of $\bar{q}_{22}/\bar{r} = 0{\cdot}1$

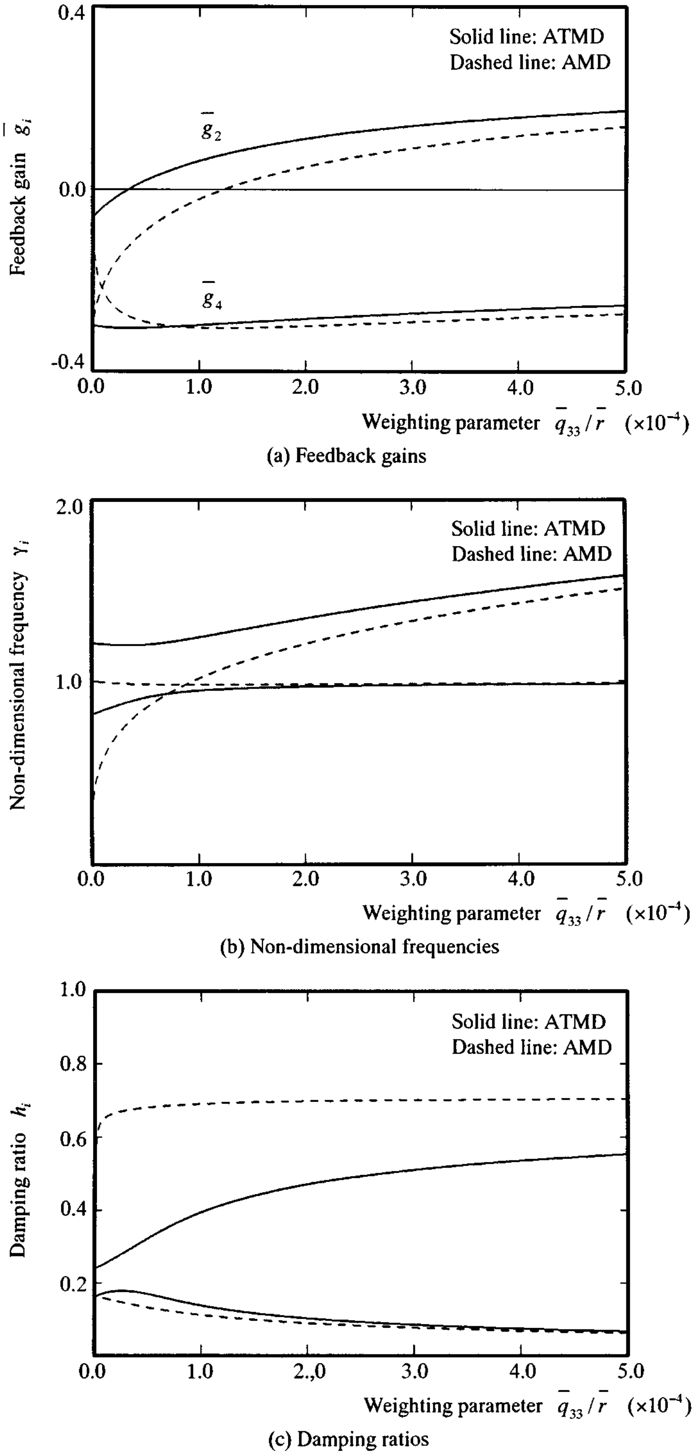

Figure 3. Magnification of Figure 2

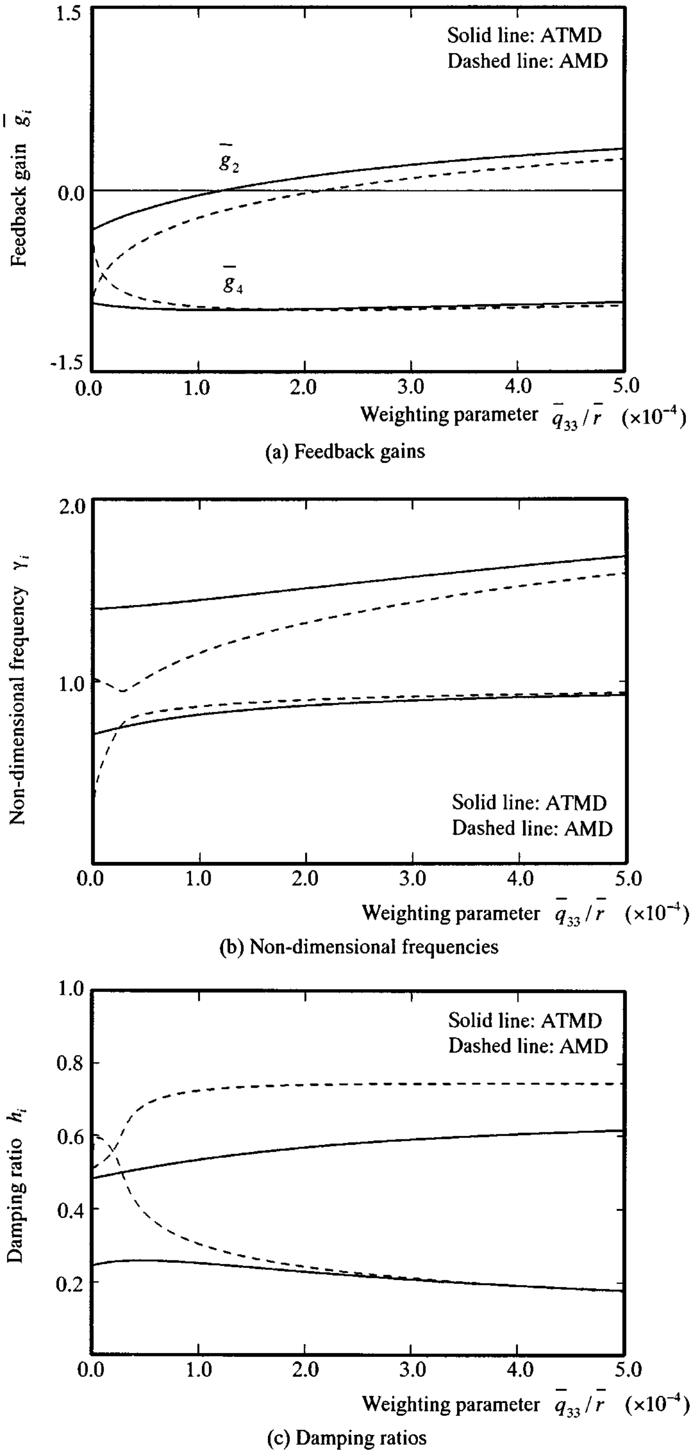

Figure 4. Feedback gains and poles depending on the weighting parameters $\bar{q}_{33}/\bar{r}$ with respect to the stroke under the condition of $\bar{q}_{22}/\bar{r} = 1{\cdot}0$

$\bar{q}_{33}/\bar{r} = 0{\cdot}256 \times 10^{-4}$. However, the AMD does not form the peak. The AMD sets the structural damping ratio to 16·4 per cent by neglecting the stroke.

Similarly, Figure 4 indicates how the feedback gains and the poles depend on the weighting parameter $\bar{q}_{33}/\bar{r}$ with $\bar{q}_{22}/\bar{r} = 1{\cdot}0$. Figure 4(a) indicates larger feedback gains than Figure 3(a) because of the highly weighted structural velocity. When the stroke is neglected in weighting, $\bar{g}_2$ is $-0{\cdot}326$ and $-0{\cdot}996$ for the ATMD and the AMD, respectively. Then, $\bar{g}_4$ is $-0{\cdot}931$ and $-0{\cdot}079$ for the ATMD and the AMD, respectively. $\bar{g}_2$ changes its sign at $\bar{q}_{33}/\bar{r} = 1{\cdot}208 \times 10^{-4}$ with $\lambda = 1{\cdot}944 \times 10^{-4}$ for the ATMD and at $\bar{q}_{33}/\bar{r} = 2{\cdot}140 \times 10^{-4}$ with $\lambda = 2{\cdot}865 \times 10^{-4}$ for the AMD. Compared with Figure 3(a), the larger weighting parameter for the structural velocity delays the change of the sign on weighting the stroke. Figure 4(b) indicates that a large $\bar{q}_{22}/\bar{r}$ makes the natural frequencies not encounter what even for the AMD. However, the two natural frequencies are caused to approach each other at $\bar{q}_{33}/\bar{r} = 0{\cdot}28 \times 10^{-4}$ by moving the structural frequency from 1·021 to 0·949, and the poles are exchanged between the damper and the structure. Because one dashed line starts with the AMD's frequency of 0·196 and approaches the structural frequency of 0·939, while another starts with the structural frequency of 1·021 and approaches the AMD's frequency of 1·592. In addition, the initial non-dimensional natural frequencies for the ATMD are 0·709 and 1·396. The large $\bar{q}_{22}/\bar{r}$ isolates the two natural frequencies from each other. Figure 4(c) indicates the complex configuration in the curves of the damping ratios near $\bar{q}_{33}/\bar{r} = 0$, corresponding to Figure 4(b). The damping ratio of the AMD varies remarkably from 9·8 per cent to the peak value of 59·4 per cent at $\bar{q}_{33}/\bar{r} = 0.063 \times 10^{-4}$, then encounters the structural curve, and increases again to 74·6 per cent at $\bar{q}_{33}/\bar{r} = 5{\cdot}0 \times 10^{-4}$. However, the structural damping ratio varies from 50·9 to 17·8 per cent, passing through the peak value of 55·6 per cent at $\bar{q}_{33}/\bar{r} = 0{\cdot}198 \times 10^{-4}$. The ATMD's damping ratio increases from 48·3 to 61·7 per cent, while the structural damping decreases from 24·5 to 17·9 per cent.

3.2. *The optimal weighting parameter with respect to an ATMD stroke*

Figures 3(c) and 4(c) show that the damping ratio forms the peak in the structural mode at a certain weighting parameter with respect to the stroke when the ATMD is installed. Figure 5 indicates the apparent damping ratio of the structural mode versus the weighting parameter $\bar{q}_{33}/\bar{r}$ when μ is 0·01 and the structural velocity is constantly weighted ($\bar{q}_{22}/\bar{r}$ is constant). From a veiwpoint of stability, the weighting parameter giving the maximum damping ratio is defined as the optimal weighting one. The optimal weighting parameter $(\bar{q}_{33}/\bar{r})_{\mathrm{opt}}$ becomes larger as the weighting parameter $\bar{q}_{22}/\bar{r}$ becomes larger. In case of $\bar{q}_{22}/\bar{r} = 0{\cdot}1$,

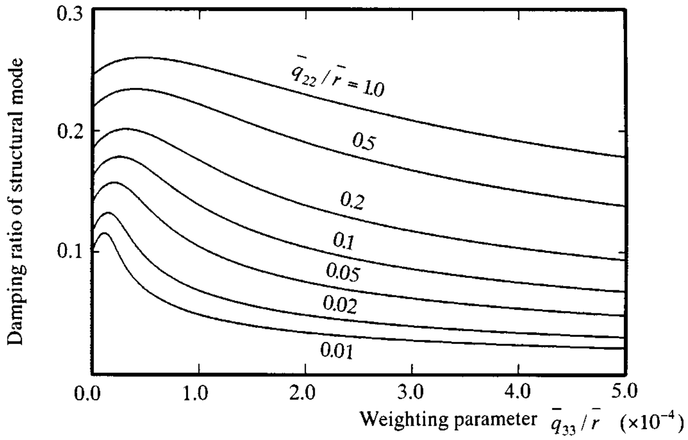

Figure 5. Damping ratio of the structural mode depending on the weighting parameter $\bar{q}_{33}/\bar{r}$ with respect to the ATMD stroke when the structural velocity is constantly weighted

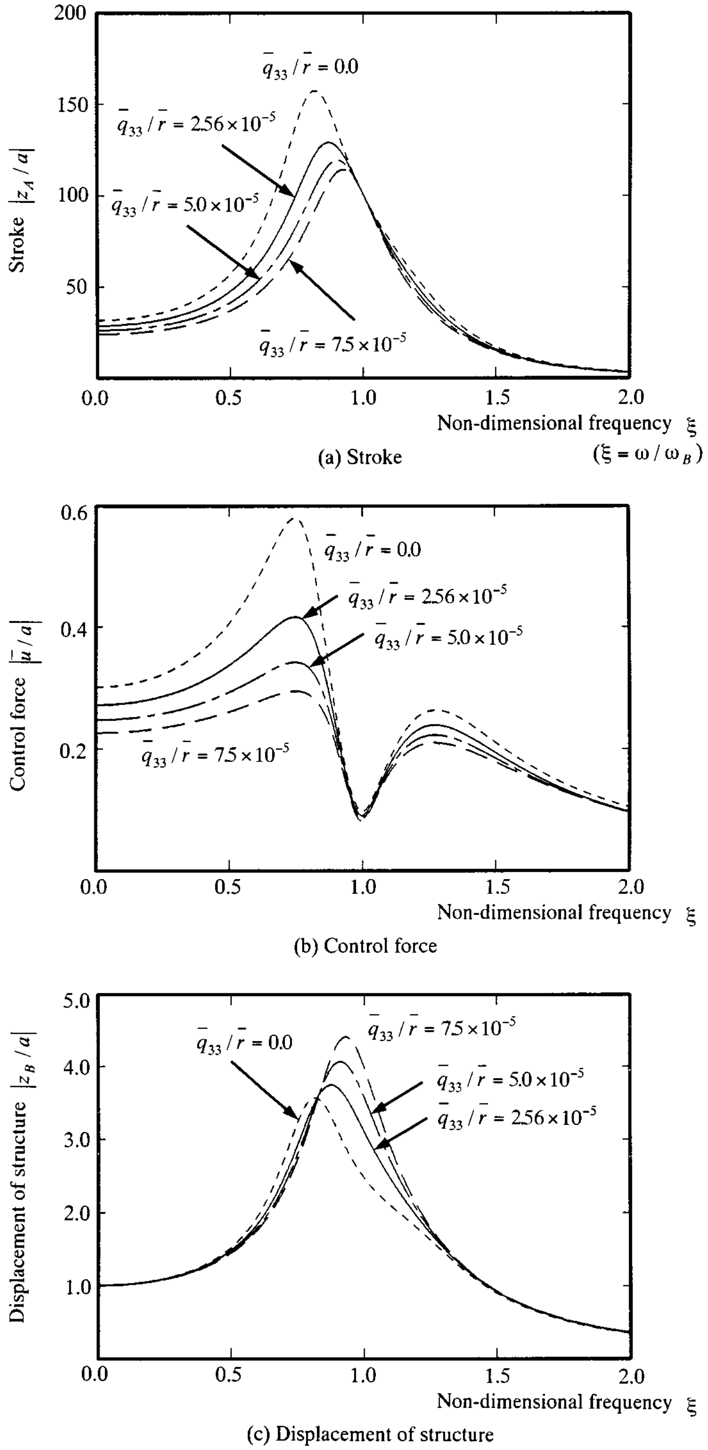

Figure 6. Frequency transfer functions of the ATMD stroke, the control force and the structural displacement to harmonic excitations, under the condition of $\bar{q}_{22}/\bar{r} = 0{\cdot}1$

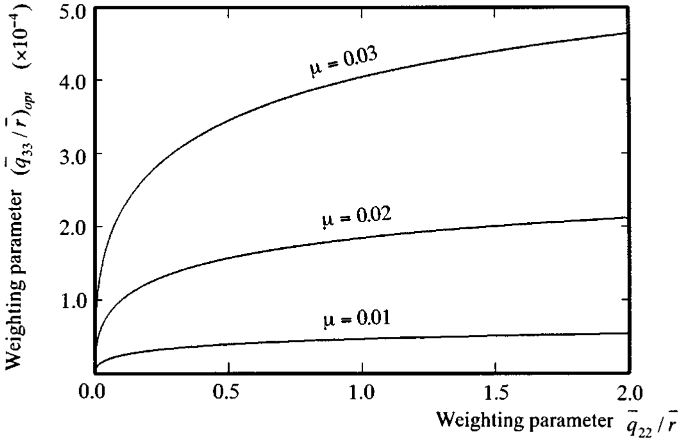

Figure 7. The optimal weighting parameter $(\bar{q}_{33}/\bar{r})_{\mathrm{opt}}$ with respect to the ATMD stroke

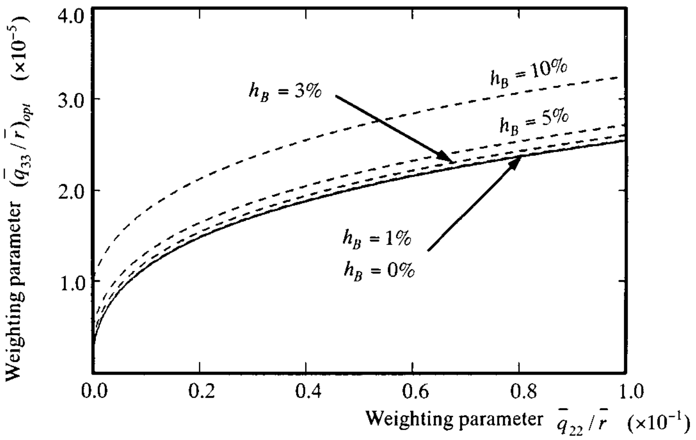

Figure 8. The optimal weighting curves depending on the structural damping ratio

the maximum structural damping ratio 17·9 per cent is larger by 1·7 per cent than when the stroke is neglected in weighting. Figure 6 illustrates the frequency transfer functions of the stroke, the control force and the structural displacement to harmonic ground excitations. Here, a is a maximum amplitude of the ground acceleration and $\xi = \omega/\omega_{\mathrm{B}}$ is a non-dimensional frequency of the excitation. Compared with the case of $\bar{q}_{33}/\bar{r} = 0{\cdot}0$, the optimal weighting parameter $(\bar{q}_{33}/\bar{r})_{\mathrm{opt}} = 2{\cdot}56 \times 10^{-5}$ advantageously suppresses the ATMD stroke by 82 per cent and the control force by 72 per cent, though the displacement of the structure shows an increase of 5 per cent. The transfer functions nearly correspond to the maximum responses when a non-stationary earthquake excitation is input.

Figure 7 indicates the optimal weighting parameter $(\bar{q}_{33}/\bar{r})_{\mathrm{opt}}$ versus $\bar{q}_{22}/\bar{r}$ when the mass ratio of the ATMD is assumed to be 0·01, 0·02 or 0·03. $(\bar{q}_{33}/\bar{r})_{\mathrm{opt}}$ becomes larger as the mass ratio becomes larger, and the gradient of the optimal weighting curve becomes gentler as $\bar{q}_{22}/\bar{r}$ increases. Figure 8 investigates how the initial damping ratio of the structure affects the optimal weighting curve when both the ATMD stroke and the structural velocity are weighted. If the initial structural damping ratio is within 5 per cent, the optimal curve can be approximated to that for undamped structure. Both the existence of the optimal weighting parameter and the sensitivities to the varieties of the design parameters are similarly confirmed when the ATMD velocity is considered in weighting.

4. CONCLUSIONS

The effect of weighting on the LQR problem is described when an SDOF undamped structure is controlled by an AMD/ATMD. The control law is introduced in a closed form after both the state equation and the performance index are non-dimensionalized. The stationary feedback gains are expressed as a function of a solution of the fourth-ordered algebraic scalar equation. The closed expression evidently indicates that the feedback gain associated with the stroke relates only to its weighting parameter and the mass and frequency ratios of the AMD to the structure, independently of the solution of the algebraic equation. In addition, the gain becomes zero when the stroke is neglected in weighting. From a broad viewpoint, weighting the stroke apparently makes the AMD damping higher toward about 70 per cent with its higher natural frequency, while it makes the structural damping lower with its nearly constant frequency. This pole allocation leads to ineffective control, which compels the auxiliary mass to become fixed to the structure. The essential feedback gain associated with the structural velocity changes its sign according to the weighting state, which becomes the important property in designing the controller. The effect of tuning the AMD to the structural natural frequency is recognized when the stroke is weighted a little.

The closer investigation indicates that the damping ratio of the structural mode apparently becomes highest at a certain small weighting parameter with respect to the ATMD stroke. With the highest damping ratio in the mode of the structure, the optimal weighting parameter suppresses both the stroke and the control force better than when only the structural velocity is weighted. The optimization for weighting the stroke is insensitive to the initial structural damping ratio if the ratio is within several per cent of the critical value.

REFERENCES

1. Y. Fujino, 'Recent research and developments on control of bridges under wind and traffic excitations in Japan', *Proc. Int. Workshop Struct. Control*, 1993, pp. 144–150.
2. T. Kobori, 'Future direction on research and development of seismic-response-controlled structure', *Proc. 1WCSC* **1**, 1994 Panel, pp. 19–31.
3. T. Kobori, N. Koshika, K. Yamada and Y. Ikeda, 'Seismic-response-controlled structure with active mass driver system. Part 2: verification', *Earthquake Engng. Struct. Dyn.* **20**, 151–166 (1991).
4. T. Fujita, 'Hybrid mass dampers with convertible active and passive modes for vibration control of tall buildings', *Proc. Int. Workshop Struct. Control*, 1993, pp. 151–166.
5. S. Okumura, I. Shimoda, N. Kawai, K. Inaba, M. Kurimoto, K. Yasui and M. Mochimaru, 'Development and verification of active/passive mass damper', *Proc. 1WCSC* **1**, 1994, WP2 pp. 73–79.
6. K. Tamura, K. Shiba, Y. Inada and A. Wada, 'Control gain scheduling of a hybrid mass damper system against wind response of a tall building', *Proc. 1WCSC* **3**, 1994, FA2 pp. 13–22.
7. T. Suzuki, M. Kageyama, A. Nobata, S. Inaba and O. Yoshida, 'Active vibration control system installed in a high-rise building', *Proc. 1WCSC* **3**, 1994, FP3 pp. 3–11.
8. T. Suzuki, M. Kageyama, A. Nobata, H. Okada and H. Fukui, 'Active vibration control system for high-rise buildings. Part 14: Vibration test on variable feedback gain control', Summaries Tech. Paper of Annual Meeting *AIJ Struct*. **1**, 753–754 (1993) (in Japanese).
9. I. Nagashima, S. Nishiyama and Y. Shinozaki, 'Response control of buildings with active mass damper. Part 2: Study on design method of variable gain feedback controller considering constraint on AMD system and its performance', *Trans. AIJ* **483**, 39–49 (1996) (in Japanese).
10. J. E. Potter, 'Matrix quadratic solutions', *SIAM j. appl. math.* **14**(3), 496–501 (1996).
11. A. J. Laub, 'A Shur method for solving algebraic Riccati equations', *IEEE Trans. Automat. Control.* **AC-24(6)**, 913–921 (1979).
12. M. Abe, 'Rule-based control algorithm for active tuned mass dampers', *J. Engng. Mech.* **122**, 705–713 (1996).
13. S. Mitsuta and K. Seto, 'Optimal design method of an active vibration controller', *Trans. Japan Soc. Mech. Engng.* **57**, 393–398 (1991) (in Japanese).
14. J. P. Den Hartog, *Mechanical Vibration*, 4th edn., McGraw-Hill, New York, 1956.
15. J. C. H. Chang and T. T. Soong, 'Structural control using active tuned mass damper', *J. Engng. Mech. Div. ASCE* **106(EM6)**, 1091–1098 (1980).

